Fractions That Hide a Quadratic

QF10-V1 · storyboard · QF10 Fractions that hide a quadratic. Read each picture with its sentences.

Picture 1

Solve $\dfrac{x}{x-1} + \dfrac{1}{x-2} = 3.$

$x \neq 1, \quad x \neq 2$ a denominator can't be 0

$x(x-2) + (x-1) = 3(x-1)(x-2)$

× both denominators

$2x^2 - 8x + 7 = 0$ from $x^2 - x - 1 = 3x^2 - 9x + 6$

$\boxed{x = 2 \pm \frac{\sqrt{2}}{2}}$ neither is 1 or 2

1. Solve x over x minus 1, plus 1 over x minus 2, equals 3.
2. x can't be 1 or 2: a denominator would be 0.
3. Multiply every term by x minus 1, times x minus 2.
4. The right side too: 3 becomes 3 times x minus 1, times x minus 2.
5. Multiply out both sides, and collect everything on one side: 2 x squared minus 8 x plus 7 equals 0.
6. The formula gives x equals 2, plus or minus root 2 over 2.
7. Check: neither is 1 or 2, so both are solutions.

Picture 2

Solve $\dfrac{2}{x-3}+\dfrac{x}{x+1}=\dfrac{8}{(x-3)(x+1)}$.

$$2(x+1)+x(x-3)=8$$ × both denominators

$$x^2-x-6=0,\quad (x-3)(x+2)=0$$

$\boxed{x=-2}$ $x=3$ makes a denominator 0

8. Now solve 2 over x minus 3, plus x over x plus 1, equals 8 over x minus 3, times x plus 1.
9. Multiply every term by x minus 3, times x plus 1.
10. On the right, that leaves just 8.
11. That gives x squared minus x minus 6 equals 0, so x is 3 or negative 2.
12. But 3 is not allowed: it makes a denominator 0.
13. Throw it away.
14. The only solution is x equals negative 2.

Picture 3

Trap $\dfrac{x}{x-1} + \dfrac{1}{x-2} = 3$

$\times \quad x(x-2) + (x-1) = 3$

$\checkmark \quad x(x-2) + (x-1) = 3(x-1)(x-2)$

15. Watch for the trap.
16. Don't multiply only the fractions: multiply every term, the 3 too.

Picture 4

Trap $(x-3)(x+2) = 0$

$\times \quad x = 3 \text{ or } x = -2$

$\checkmark \quad x = -2$

17. And don't keep a value that makes a denominator 0.