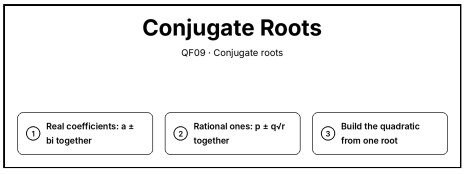
Conjugate Roots: teacher script

QF09-V1 · QF09 Conjugate roots · teachers only. Without a screen, read each Say line and draw each Draw step on the board. About 100 seconds as a video.

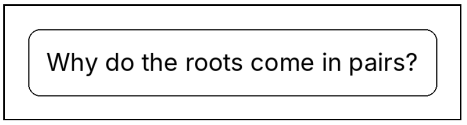
01



Say. Conjugate roots. With real coefficients, roots that aren't real come in pairs. With rational coefficients, so do roots with a square root in them. And from one root you can build the whole quadratic.

Draw. Write the title “Conjugate Roots” and the steps: 1. Real coefficients: a ± bi together; 2. Rational ones: p ± q√r together; 3. Build the quadratic from one root.

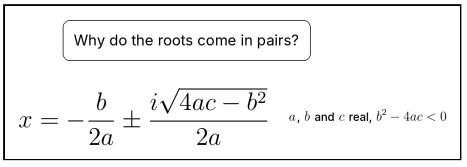
02



Say. Why do the roots come in pairs? Look at the formula when b squared minus 4 a c is negative.

Draw. Clear the board. Write “Why do the roots come in pairs?” in a box at the top.

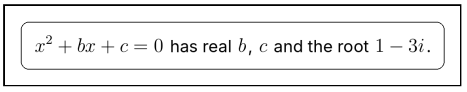
03



Say. Both roots have the same real part, negative b over 2 a, and imaginary parts with opposite signs. They are conjugates.

Draw. Write x = −b/(2a) ± \dfraci√(4ac − b²)2a on the next line. Beside it, write “a, b and c real, b² − 4ac < 0”.

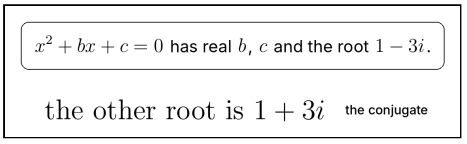
04



Say. x squared plus b x plus c equals 0, with b and c real, has the root 1 minus 3 i. Find b and c.

Draw. Clear the board. Write “x² + bx + c = 0 has real b, c and the root 1 − 3i.” in a box at the top.

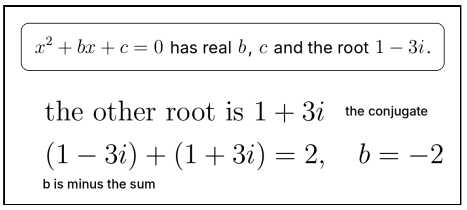
05



Say. The coefficients are real, so the other root is its conjugate: 1 plus 3 i.

Draw. Write the other root is 1 + 3i on the next line. Beside it, write “the conjugate”.

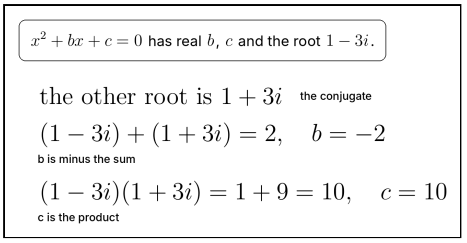
06



Say. The roots add up to 2, and b is minus the sum: b is negative 2.

Draw. Write (1 − 3i) + (1 + 3i) = 2, b = −2 on the next line. Beside it, write “b is minus the sum”.

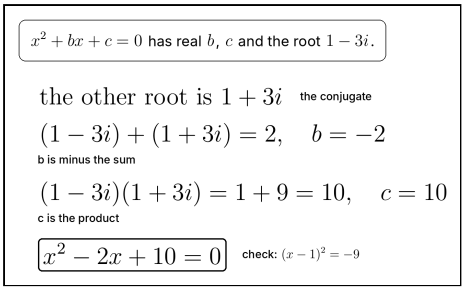
07



Say. The product is 1 minus 9 i squared, which is 1 plus 9, or 10. So c is 10.

Draw. Write (1 − 3i)(1 + 3i) = 1 + 9 = 10, c = 10 on the next line. Beside it, write “c is the product”.

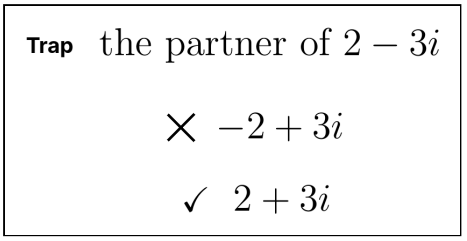
08



Say. The quadratic is x squared minus 2 x plus 10. Check: completing the square gives x minus 1, all squared, equals negative 9, so x is 1 plus or minus 3 i.

Draw. Write x² − 2x + 10 = 0 and box it on the next line. Beside it, write “check: (x − 1)² = −9”.

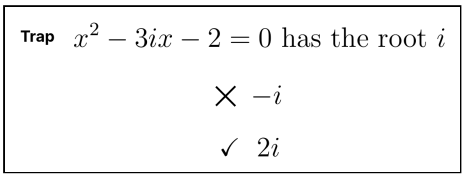
09



Say. Watch for the trap. Don't change the real part: change the sign of only the imaginary part.

Draw. Clear the board. Write “Trap” and the partner of 2 − 3i. Below, write −2 + 3i and cross it out ×. Then write 2 + 3i with a tick ✓.

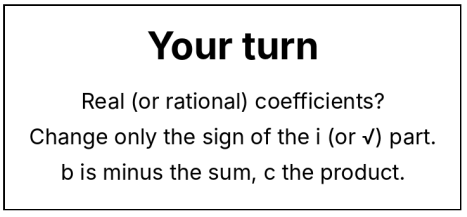
10



Say. And the rule needs real coefficients. Here the coefficient of x isn't real, so the other root isn't negative i. The roots add up to 3 i, so it is 2 i.

Draw. Clear the board. Write “Trap” and x² − 3ix − 2 = 0 has the root i. Below, write −i and cross it out ×. Then write 2i with a tick ✓.

11



Say. Now it is your turn. Check that the coefficients are real, or rational. Change the sign of only the i part, or the root part. Then b is minus the sum, and c is the product.

Draw. Point to today’s activity: Real (or rational) coefficients? Change only the sign of the i (or √) part. b is minus the sum, c the product.