Conjugate Roots

QF09-V1 · storyboard · QF09 Conjugate roots. Read each picture with its sentences.

Picture 1

Why do the roots come in pairs?

$$x = -\frac{b}{2a} \pm \frac{i\sqrt{4ac - b^2}}{2a} \qquad a, b \text{ and } c \text{ real, } b^2 - 4ac < 0$$

1. Why do the roots come in pairs?
2. Look at the formula when b squared minus 4 a c is negative.
3. Both roots have the same real part, negative b over 2 a, and imaginary parts with opposite signs.
4. They are conjugates.

Picture 2

$x^2 + bx + c = 0$ has real b, c and the root $1 - 3i$.

$$\text{the other root is } 1 + 3i$$ the conjugate

$$(1 - 3i) + (1 + 3i) = 2, \quad b = -2$$

b is minus the sum

$$(1 - 3i)(1 + 3i) = 1 + 9 = 10, \quad c = 10$$

c is the product

$$\boxed{x^2 - 2x + 10 = 0}$$ check: $(x - 1)^2 = -9$

5. x squared plus b x plus c equals 0, with b and c real, has the root 1 minus 3 i.
6. Find b and c.
7. The coefficients are real, so the other root is its conjugate: 1 plus 3 i.
8. The roots add up to 2, and b is minus the sum: b is negative 2.
9. The product is 1 minus 9 i squared, which is 1 plus 9, or 10.
10. So c is 10.
11. The quadratic is x squared minus 2 x plus 10.
12. Check: completing the square gives x minus 1, all squared, equals negative 9, so x is 1 plus or minus 3 i.

Picture 3

Trap $\text{the partner of } 2 - 3i$

$\times \ -2 + 3i$

$\checkmark \ \ 2 + 3i$

13. Watch for the trap.
14. Don't change the real part: change the sign of only the imaginary part.

Picture 4

Trap $x^2 - 3ix - 2 = 0 \text{ has the root } i$

$\times \ -i$

$\checkmark \ \ 2i$

15. And the rule needs real coefficients.
16. Here the coefficient of x isn't real, so the other root isn't negative i.
17. The roots add up to 3 i, so it is 2 i.