Sum and Product, Again

QF08-V1 · storyboard · QF08 Sum and product, again. Read each picture with its sentences.

Picture 1

Add and multiply the two roots of $ax^2 + bx + c = 0$.

$$\frac{-b+\sqrt{D}}{2a} + \frac{-b-\sqrt{D}}{2a} = -\frac{b}{a}$$

$D = b^2 - 4ac$: the roots cancel

$$\frac{-b+\sqrt{D}}{2a} \cdot \frac{-b-\sqrt{D}}{2a} = \frac{b^2 - D}{4a^2} = \frac{c}{a}$$

$b^2 - D = 4ac$

1. Add and multiply the two roots of a x squared plus b x plus c equals 0.
2. Write D for b squared minus 4 a c.
3. Add the two roots from the formula and the roots cancel: negative 2 b over 2 a, which is negative b over a.
4. Multiply them: b squared minus D, over 4 a squared.
5. b squared minus D is 4 a c, so the product is c over a.

Picture 2

Roots of $2x^2 - 4x - 7 = 0$: $1 \pm \frac{3\sqrt{5}}{2}$, $1 \pm \frac{3\sqrt{2}}{2}$ or $1 \pm \frac{3\sqrt{3}}{2}$?

$$r + s = -\frac{-4}{2} = 2, \quad rs = -\frac{7}{2}$$ signs kept

$$(1+p)(1-p) = 1 - p^2 = -\frac{7}{2}$$ use the product

$$\boxed{p^2 = \frac{9}{2}, \quad x = 1 \pm \frac{3\sqrt{2}}{2}}$$ $\left(\frac{3\sqrt{2}}{2}\right)^2 = \frac{18}{4} = \frac{9}{2}$

6. Which are the roots of 2 x squared minus 4 x minus 7 equals 0?
7. Three pairs to choose from, and no solving.
8. The roots add up to 2, and multiply to negative 7 halves.
9. Every pair adds up to 2, so use the product: 1 plus p, times 1 minus p, is 1 minus p squared.
10. So p squared is 9 halves, and 3 root 2 over 2, squared, is 9 halves.
11. The roots are 1 plus or minus 3 root 2 over 2.

Picture 3

Trap $2x^2 - 4x - 7 = 0$

$\times\ r + s = -2$

$\checkmark\ \ r + s = 2$

12. Watch for the trap.
13. Mind the minus: the sum is negative b over a, and b is negative 4.

Picture 4

Trap $2x^2 - 4x - 7 = 0$

$\times\ rs = -7$

$\checkmark\ \ rs = -\frac{7}{2}$

14. And the product isn't negative 7: divide by a.
15. The product is c over a.