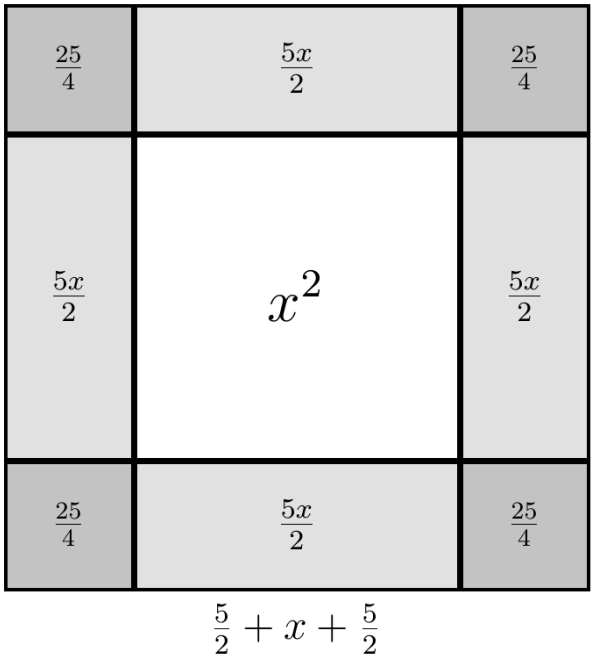
Complete the Square

QF03-V1 · storyboard · QF03 Complete the square. Read each picture with its sentences.

Picture 1



$$x^2 + 10x = 39$$

Corners: $4 \times \frac{25}{4} = 25$

Big square: $39 + 25 = 64$

Side: $x + 5 = 8$, so $x = 3$

1. About twelve hundred years ago, al-Khwarizmi solved x squared plus 10 x equals 39 with a picture.
2. Start with a square of side x.
3. Put four rectangles round it, each x by 5 halves: together they make 10 x, so the whole shape has area 39.
4. Now fill in the four corners.
5. Each is 5 halves by 5 halves, and the four together make 25, so add 25 to both sides.
6. The big square has area 64, so its side is 8.
7. That side is x plus 5, so x is 3.
8. He missed the second root, negative 13: a negative side made no sense to him.

Picture 2

Solve $x^2 + 2x - 7 = 0.$

$$x^2 + 2x = 7$$ move the 7 across

$$x^2 + 2x + 1 = 7 + 1$$ half of 2, squared, on both sides

$$(x + 1)^2 = 8$$

$$x + 1 = \pm 2\sqrt{2} \quad \sqrt{8} = 2\sqrt{2}$$

$$\boxed{x = -1 \pm 2\sqrt{2}}$$

9. Now with symbols.
10. Solve x squared plus 2 x minus 7 equals 0.
11. Move the 7 across: x squared plus 2 x equals 7.
12. Half of 2 is 1, and 1 squared is 1.
13. Add 1 to both sides.
14. The left side is x plus 1, all squared, and it equals 8.
15. Undo the square: x plus 1 equals plus or minus root 8, which is 2 root 2.
16. Subtract 1: x equals negative 1, plus or minus 2 root 2.

Picture 3

Trap $x^2 + 2x + 1 = 7$

$\times \quad (x+1)^2 = 7$

$\checkmark \quad (x+1)^2 = 8$

17. Watch for the trap.
18. Add the 1 to one side only, and every root after it is wrong.
19. Add it to both sides.